Advantages of Linear Programming Techniques

The main advantages of linear programming are given below;

1. It indicates how the available resources can be used in the best way.
2. It helps in attaining the optimum use of the productive resources and man- power.
3. It improves the quality of decisions.
4. It reflects the drawbacks of the production process.
5. The necessary modifications of the mathematical solutions is also possible by using Linear Programming.
6. It helps in re-evaluation of a basic plan with changing conditions.

Solution of a LPP

In general, we use the following methods for the solution of a LPP;

1. Graphical Method
2. Simplex Method
   (i) Big-M Method
   (ii) Two-Phase Method

GRAPHICAL METHOD

Introduction

If the objective function Z is a function of two variables only the problem can be solved by graphical method. A problem of three variables can be also solved by this method but it is complicated.

Procedures for Solving LPP by Graphical Method

Various procedures of solving LPP by graphical method are as follows;

1. Formulation of the problem into LPP model
   The problem is expressed in the form of a mathematical model.

Here the objective function and the constraints are written down.

2. Consider each inequality constraints as equation.

3. Plot each equation on the graph as each equation will geometrically represent a straight line.

4. Shade the feasible region and identify the feasible solutions
   Every point on the line will satisfy the equation of line. If the inequality constraints corresponding to that line is $\leq$ then the region below the line lying in the first quadrant (due to non-negativity of variables) is shaded. For the inequality constraints with $\geq$ sign the region above the line in the first quadrant is shaded. The point lying in common region will satisfy all the constraints simultaneously. Thus, the common region obtained is called feasible region. This region is the region of feasible solution. The corner points of this region are identified.

5. Finding the optimal solutions
   The value of Z at various corners points of the region of feasible solution are calculated. The optimum (maximum or minimum) Z among these values is noted. Corresponding solution is the optimal solution.

Note: While finding the corner points, greater accuracy is needed, the ordinates may be obtained by algebraically solving the corresponding equations.

Example 1.

Solve the following LPP graphically

$$Max(Z) = 3x_1 + 5x_2$$

Subject to constraints

$$x_1 + 2x_2 \leq 2000$$
$$x_1 + x_2 \leq 1500$$
$$x_2 \leq 600$$

and

$$x_1, x_2 \geq 0$$

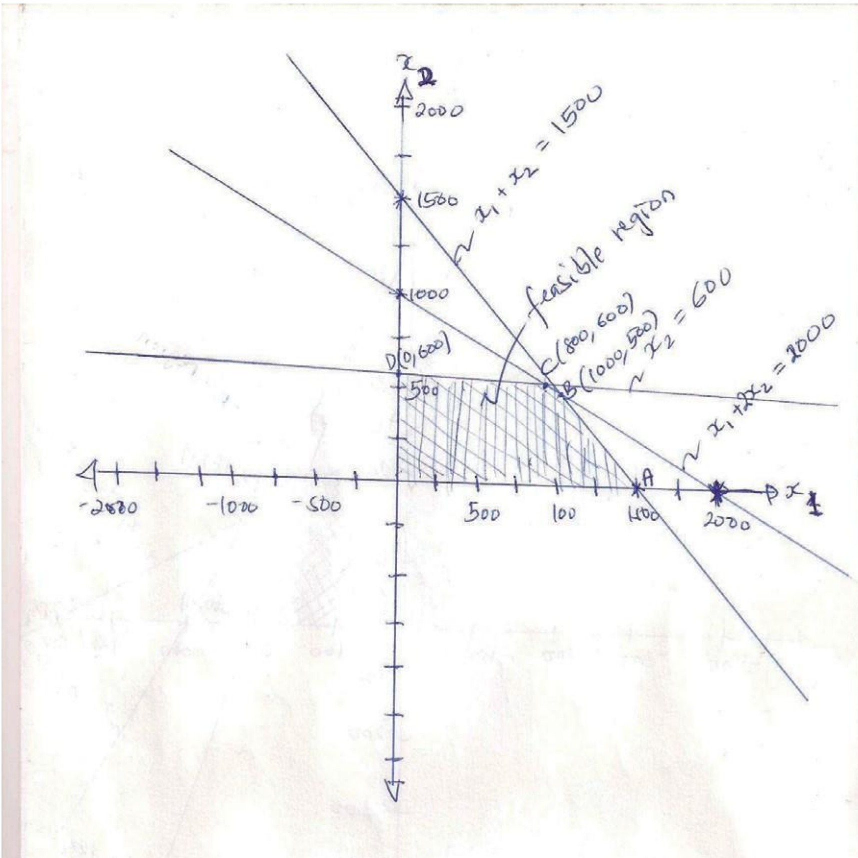
Solution:

To represent the constraints graphically the inequalities are written as equalities. Every equation is represented by a straight line.

To draw the lines, two points on each of the lines are found as indicated in the following table (intercepts);

Equation	x_2 intercept when $x_1 = 0$	x_1 intercept when $x_2 = 0$	Point (x, y) on the line
$x_1 + 2x_2 = 2000$	$x_2 = 1000$	$x_1 = 2000$	(0,1000)(2000,0)
$x_1 + x_2 = 1500$	$x_2 = 1500$	$x_1 = 1500$	(0,1500)(1500,0)

$x_2 = 600$ and $x_1 = 0$, x_1 axis $x_2 = 0$, x_2 axis. Plot each equation on the graph.



B and C are the point of intersection of lines $x_1 + 2x_2 = 2000$, $x_1 + x_2 = 1500$ and $x_1 + 2x_2 = 2000$, $x_2 = 600$ on solving we get $B = (1000, 500)$,$C = (800, 600)$

Corner Points	Value of $Z = 3x_1 + 5x_2$
$A(1500, 0)$	4500
$B(1000, 500)$	5500(Max. Value)
$C(800, 600)$	5400
$D(0, 600)$	3000

Therefore, the Maximum value of Z occurs at B (1000, 500), hence the optimal solution is $x_1 = 1000$ and $x_2 = 500$.

Example 2:
A and B are two product to be manufactured, unit profits are Rs. 40 and Rs. 35 respectively. Maximum materials available are 60 kgs and 96 hrs. Each units of A needs 2 kg of materials and 3 man-hours, whereas each units of B needs 4 kg of materials and 3 man-hours. Find optimal level of A and B to be manufactured.

Solution:

First of all we have to formulate the model as follows;

Decision Variables:

Let x_1 and x_2 be the number of product *A* and *B* to be produced by the manufacturer respectively.

Objective Functions:

Therefore, the total profit (in Rs.) has to be maximized.

$$Max(Z) = 40x_1 + 35x_2$$

Constraints

The manufacturer has limited materials and labour time for manufacturing product A and B. There are 60 kgs of material available and 96 hrs available for labour. Thus, total processing is restricted.

$$2x_1 + 3x_2 \leq 60, \textit{Material Constraint}$$
$$4x_1 + 3x_2 \leq 96, \textit{Labour time Constraint}$$

Finally the complete LPP is;

$$Max(Z) = 40x_1 + 35x_2$$

Subject to

$$2x_1 + 3x_2 \leq 60$$
$$4x_1 + 3x_2 \leq 96$$

and

$$x_1, x_2 \geq 0$$

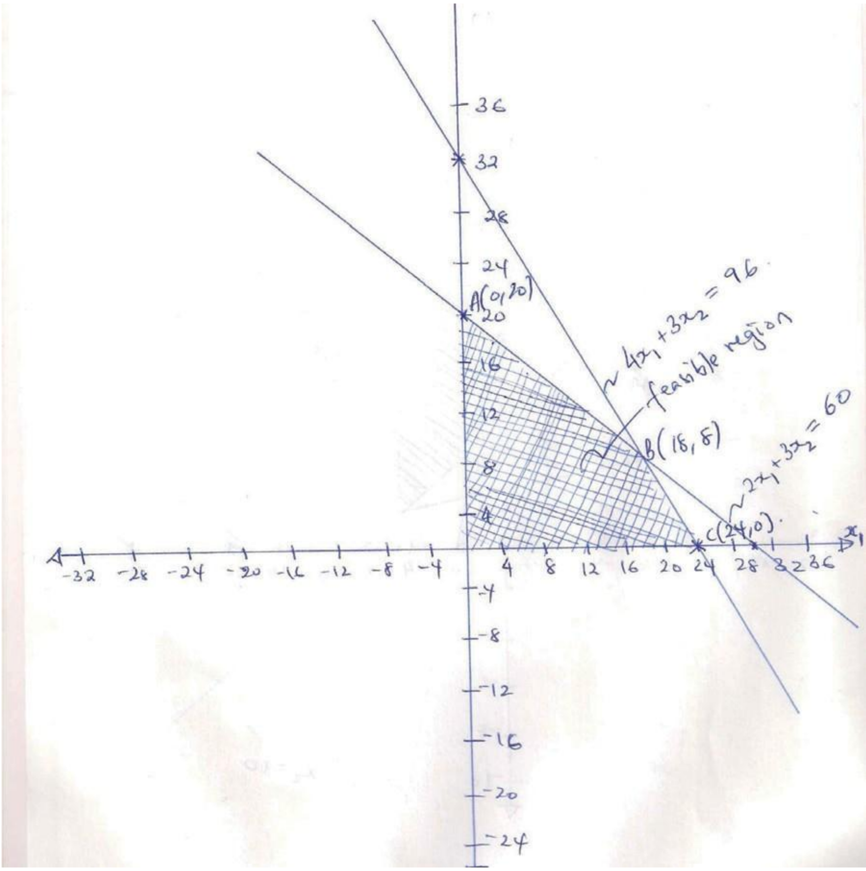
To represent the constraints graphically the inequalities are written as equalities.

Every equation is represented by a straight line.

To draw the lines, two points on each of the lines are found as indicated in the following table (intercepts);

Equation	x_2 intercept when $x_1 = 0$	x_1 intercept when $x_2 = 0$	Point (x, y) on the line
$2x_1 + 3x_2 = 60$	$x_2 = 20$	$x_1 = 30$	(0,20)(30,0)
$4x_1 + 3x_2 = 96$	$x_2 = 32$	$x_1 = 24$	(0,32)(24,0)

and $x_1 = 0$, x_1 axis $x_2 = 0$, x_2 axis. Plot each equation on the graph.



B is the point of intersection of lines $2x_1 + 3x_2 = 60$ and $4x_1 + 3x_2 = 96$ on solving we get $B = (18, 8)$

Corner Points	Value of $Z = 40x_1 + 35x_2$
$A(0, 20)$	700
$B(18, 8)$	1000(Max. Value)
$C(24, 0)$	960

Therefore, the Maximum value of Z occurs at $B(18,8)$, hence the optimal solution is $x_1 = 18$ and $x_2 = 8$.